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# Structural Contagion and Vulnerability to Unexpected Liquidity Shortfalls

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## Abstract

This paper assumes that financial fluctuations are the result of the dynamic interaction between liquidity and solvency conditions of individual economic units. The framework is an extension of Sordi and Vercelli (this issue) designed as an heterogeneous agent model which proceeds through discrete time steps within a finite time horizon. The interaction at the micro-level between economic units monitors the spread of contagion and systemic risk, producing interesting complex dynamics. The model is analyzed by means of numerical simulations and systemic risk modelling, where local interaction of units is captured and analysed by the bilateral provision of liquidity among units. The behavior and evolution of economic units are studied for different parameter regimes in order to investigate the relation between units' expectations, liquidity regimes and contagion. Liquidity policy implications are briefly discussed.

*Keywords:* Financial fluctuations, Contagion, Systemic risk, Heterogeneous agents, Complex dynamics

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## 1. Introduction

The complex dynamics of financial fluctuations reflects the impact of individual decisions on the macro variables of the economy within the technological, financial and institutional constraints characterizing a given economy.

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5 Among the structural constraints the financial conditions of economic units  
6 play a crucial role. In particular, the interaction between the liquidity and  
7 solvency conditions of economic units seems to play a crucial role in the emer-  
8 gence of complex financial fluctuations. This interaction is significant at the  
9 level of single financial units, under suitable simplifying assumptions, to mod-  
10 elling the typical fluctuations of their financial conditions. The model may  
11 be also applied to the economy as a whole providing some general insights on  
12 its financial fluctuations and its policy implications (Sordi and Vercelli, this  
13 issue, 2006; Vercelli, 2000, 2011; Dieci et al., 2006). However, a crucial fea-  
14 ture of real financial fluctuations is missing in this approach: the interaction  
15 between the units’ balance sheets that heavily affects their behavior.

16 A case in point is the so-called “contagion” that has played a crucial role  
17 in recent financial crises and has enhanced systemic risk.<sup>2</sup> There is a vast  
18 and growing literature on systemic risk and financial contagion, in particular  
19 post financial crisis 2007-2008 (see Haldane (2009) for an overview). The  
20 origins of systemic risk has been modelled as a “bank run” where the loss in  
21 confidence in the banking system pushes depositors to withdraw money from  
22 their banks, so triggering a contagion effect (see for example De Bandt and  
23 Hartmann (2000) for an overview of different forms of systemic risk). Alter-  
24 natively, it can occur as a consequence of an exogenous shock (see Kaufman  
25 and Scott (2003); Kaufmann (2005) for a qualitative overview).

26 The other important component of systemic risk modelling is the descrip-  
27 tion of the spread of contagion in the system. Two main approaches have  
28 been developed in recent literature. The first one is what it is called the “fire-  
29 sale” approach, in which the propagation of contagion is driven by the losses  
30 in asset values due to liquidation of failing banks (Allen and Gale, 2001; Dia-  
31 mond and Rajan, 2005; Freixas et al., 2000). The second one directly models  
32 the bilateral flows between units in a financial network as a direct channel of  
33 contagion (see Markose et al. (this issue); Krause and Giansante (this issue);  
34 Vivier-Lirimont (2004); Iori et al. (2008); Cohen-Cole et al. (2010) for an  
35 extensive literature review).

36 This work will address the problem of systemic risk by modelling the ori-  
37 gins of contagion as a result of an endogenous liquidity shock closely related

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<sup>2</sup>According to the Bank of International Settlements, systemic risk is defined as “the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties” (BIS, 1994)

to the “bank run” approach, in which the trigger event is driven by the economic business cycle as described in Sordi and Vercelli (this issue) and the central bank liquidity policy. The losses spread along direct connections of liquidity flows between units in the network of liquidity provision.

The interaction between units’ balance sheets is particularly visible during the most serious financial crises but plays a crucial role in all the phases of the cycle as it reflects the interdependence via the financial inflows and outflows of the economic units. Increased outflows of a unit translate in increased inflows of other units and the other way round. In order to study this interaction we adopt an heterogeneous-agent model which proceeds through discrete time steps within a finite time horizon. Economic units decide their liquidity flows on the basis of their liquidity index, solvency index and financial fragility parameters, defined in the next Section. It is also assumed that the units can also borrow money from the Central Bank in order to meet their liquidity needs.

The paper is organized as follows. Section 2 presents the approach explored in this paper by modelling the behavior of financial units and their interactions with other units and the Central Bank. Section 3 presents an exercise of a stylized economy with three units interacting under different liquidity policy regimes. Section 4 extends the previous results by modelling and simulating a larger system. Section 5 concludes.

## 2. The Model

The model proceeds through discrete time steps. A set of  $N$  economic units (each labelled by an index  $i = 1, \dots, N$ ) interacts at each timestep  $t$ . Economic units encompass all kinds of private economic units including firms, banks, and households. The private units of a certain economy interact not only among themselves but also with the public economic units. We will consider only one of the public units that plays a crucial role in monetary and financial policy: the Central Bank.

### 2.1. Current Financial Ratios

Units are heterogeneous in terms of their liquidity positions and exchange strategies.

Let us define an  $N \times N$  matrix  $X$  of bilateral flows where the element  $x_{i,j}$  represents the directed flow between units  $i$  and  $j$ . This flow is an inflow from the point of view of unit  $j$  and an outflow from the point of view of

unit  $i$ . Therefore, the elements of the row  $i$  describe the outflows of the unit  $i$ , while the elements of the column  $j$  describes the inflows of the unit  $j$ .  
The sum of the elements of the row  $i$  equals the total outflow of unit  $i$ , while the sum of the elements of the column  $j$  equals the total inflos of unit  $j$ .

$$y_{j,t} = \sum_i x_{ji}, \quad (1)$$

and

$$e_{i,t} = \sum_j x_{ij}. \quad (2)$$

define the total inflows and outflows respectively.

We measure the *liquidity* of unit  $i$  at time  $t$  according to Sordi and Vercelli (this issue) as the surplus of financial inflows  $y_{i,t}$  minus financial outflows  $e_{i,t}$ , all flows being measured at the same time unit. We can then define the *liquidity index* for unit  $i$  at time  $t$ ,  $f_{i,t}$ , as

$$f_{i,t} = y_{i,t} - e_{i,t}. \quad (3)$$

On the other hand, the *solvency* of the unit is captured by its “net worth”, defined as the discounted value of its expected surpluses and deficits. We define the *solvency index* for unit  $i$  at time  $t$ ,  $f_{i,t}^*$ , as

$$f_{i,t}^* = \sum_{s=0}^{T_i} \frac{E_{t-1}[f_{i,t+s}]}{(1+r)^s}. \quad (4)$$

where  $E_{t-1}[\cdot]$  denotes the conditional expectation operator based upon information available at the end of period  $t-1$  and  $T_i \geq 1$  is the time horizon of unit  $i$ . The nominal interest rate  $r$  is used as the discount factor.<sup>3</sup>

## 2.2. Dynamic interaction between units' solvency and liquidity conditions

Following Sordi and Vercelli (this issue), we obtain a feedback relation between the solvency and liquidity indices under the assumption that units forecast their future liquidity indices for the next  $T_i$  periods by using a mix

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<sup>3</sup>We do not need to distinguish between nominal and real interest rates as prices are fixed for simplicity.

94 of heterogenous (extrapolative and regressive) expectations.<sup>4</sup> In this case,  
 95 from (4) we arrive at the equation

$$f_{i,t+1} = \beta_{i,t}^{er} f_{i,t} + (1 - a_i \beta_{i,t}^{er}) f_{i,t}^* \quad (5)$$

96 where  $a_i = \sum_{s=0}^{T_i} [1/(1+r)^s] = r(1+r)^{T_i} / [(1+r)^{T_i+1} - 1]$  and where,  
 97 indicating by  $\rho_i^e$  and  $\rho_i^r$  the coefficients of extrapolative and regressive expect-  
 98 tations respectively, and with  $b$  a small positive magnitude, we have:

$$\beta_{i,t}^{er} = \begin{cases} \frac{(1-\rho_i^r)[(1+r)^{T_i+1} - (1-\rho_i^r)^{T_i+1}]}{(1+r)^{T_i}(r+\rho_i^r)}, & \text{if } |f_{i,t} - a_i \mu_i| > b \\ \frac{(1+\rho_i^e)[(1+r)^{T_i+1} - (1+\rho_i^e)^{T_i+1}]}{(1+r)^{T_i}(r-\rho_i^e)}, & \text{if } |f_{i,t} - a_i \mu_i| < b \end{cases} \quad (6)$$

99 In this definition of the parameter  $\beta_{i,t}^{er}$ , the crucial role is played by  $\mu$  that  
 100 represents a safe liquidity margin that units do not want to bridge, i.e. the  
 101 minimum value of its net worth sufficiently higher than zero, beneath which  
 102 units do not want to go.

103 Then, we define here the *desired liquidity index*  $\hat{f}_{i,t+1}$  as the liquidity  
 104 target that unit  $i$  aims to achieve at  $t+1$ . It is described by the dynamic  
 105 equation

$$\hat{f}_{i,t+1} = f_{i,t} - \alpha(f_{i,t}^* - \mu_i). \quad (7)$$

106 Following Sordi and Vercelli (this issue), when a unit at time  $t$  is aware that  
 107 the value of its solvency index is greater than the safety margin  $\mu_i$ , it reacts  
 108 (in the next period) by reducing outflows relative to inflows and vice versa  
 109 when the value of its index is less than  $\mu_i$ .

110 The intended contribution of this paper is the description and imple-  
 111 mentation of the micro-interactions between units that originate from their  
 112 attempts to meet their individual liquidity targets. As a preliminary step,  
 113 the latter are described and discussed in the next section.

### 114 2.3. Liquidity targets

115 At each time step, units compute their liquidity index  $f_{i,t}$  (3) and their  
 116 solvency index  $f_{i,t}^*$  (4) on the basis of the bilateral matrix flow  $X_t$ . Depending  
 117 on the value of their solvency index, units may decide to increase or decrease

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<sup>4</sup>We emphasize that liquidity indices  $f$  are realized values while the solvencies indices  $f^*$  are expected values.

118 their current liquidity index by modifying the level of inflows and outflows.  
 119 One way to do so is to negotiate new monetary flows with other units with  
 120 opposite liquidity needs. In order to differentiate units by their liquidity  
 121 needs, we define here the absolute deviation  $z$  of the liquidity index  $f_{i,t}$  from  
 122 the next period liquidity target  $\widehat{f}_{i,t+1}$  as follows:

$$z_{i,t} = \widehat{f}_{i,t+1} - f_{i,t}, \quad (8)$$

123 The value of  $z$  captures the absolute change of the current liquidity index  
 124 to meet the outflows targets for the next period (captured by  $\widehat{f}$ ). In other  
 125 words, the sign of  $z$  allows us to differentiate between units with a positive  
 126 and a negative liquidity gradient (i.e. units that want to increase or decrease  
 127 their liquidity index respectively).

128 It is clear that new liquidity negotiation can only occur between units  
 129 with opposite signs of  $z$ . The next section presents a matching mechanism  
 130 of bilateral liquidity flows adjustment according to each liquidity target.

#### 131 2.4. Matching mechanism

132 In principle there is an infinite number of ways to adjust  $e_{i,t+1}$  and  $y_{i,t+1}$   
 133 that satisfy  $\widehat{f}_{i,t+1}$ . In particular, the deviation  $z$  is used as a proxy for the  
 134 gradient of liquidity that the unit aims to achieve. A positive value of  $z$   
 135 describes an economic unit wishing to increase the liquidity index for the  
 136 next period, while a negative value of  $z$  describes an economic unit wishing  
 137 to reduce the liquidity index. Both positive and negative changes can be  
 138 easily achieved by modifying the outflows or inflows according to the rules of  
 139 the matching mechanism. We model a mechanism that matches units with  
 140 opposite directional changes<sup>5</sup>.

141 We define two new variables  $\tau_{i,t}^y$  and  $\tau_{i,t}^e$  as percentage change in  $y$  and  $e$ ,  
 142 consistent with  $\widehat{f}$  as

---

<sup>5</sup>The same results may be achieved by decreasing inflows (for negative  $z$ ) or outflows (for positive  $z$ ). However, this second option would require renegotiation procedures between parties about previous deals with penalties to be paid from the party who decides to withdraw from a contract. As we are only interested in the evolution of the indices (not inflow and/or outflows separately), the incremental mechanism is the easiest solution at this stage.

$$\tau_{i,t}^y = \begin{cases} \frac{z_{i,t}}{y_{i,t}} & \text{if } z_{i,t} > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

$$\tau_{i,t}^e = \begin{cases} \frac{|z_{i,t}|}{e_{i,t}} & \text{if } z_{i,t} < 0, \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

As we can see in (9) and (10),  $\tau_{i,t}^y$  takes only positive values of  $z_{i,t}$ , relative to the amount of inflow of the unit. It represents the percentage change of inflow needed to reach a positive change in liquidity. On the other side,  $\tau_{i,t}^e$  takes only negative values of  $z_{i,t}$  (in absolute terms) relative to the amount of outflows the unit has to increase in order to decrease its liquidity index (as  $z$  is negative). In other words, we use  $\tau^y$  and  $\tau^e$  to split our population of economic units into two groups on the basis of the liquidity gradient. Therefore, the two populations will try to achieve different investment decisions, either by increasing inflows ( $\tau^y$ ) or by increasing outflows ( $\tau^e$ ).

The evolution of each bilateral flow  $x_{ij}$  is computed according to

$$x_{ij,t+1} = (1 + \min [\tau_{j,t}^y, \tau_{i,t}^e]) x_{ij,t}. \quad (11)$$

Equation (11) describes a dynamic update of bilateral flows that takes into account the strategies of units  $i$  and  $j$  for next period. Remember that the flow  $x_{ij}$  is an outflow for  $i$  and an inflow for  $j$ . In the case of a matching both parties  $i$  and  $j$  agree to increase the flow  $x_{ij}$ , which can only accomodate the minimum of the two. Therefore, the *min* function in equation (11) applied to  $\tau_{i,t}^e$  and  $\tau_{j,t}^y$  (representing the percentage increase of the flow  $x_{ij}$  that agents  $i$  and  $j$  would like to achieve for next period respectively), prevents an overincrease of  $x_{ij}$  that would be above the limit of one of the two parties.

## 2.5. Central bank liquidity policy

Business cycle fluctuations directly affect the success of units in meeting their liquidity needs. However, the intervention of the Central Bank as lender/borrower of last resort can help units to fulfill their liquidity goals by fully or partially offsetting the units' position not covered by the other units.

We define the central bank intervention  $C_{i,t}$  for unit  $i$  for next time period  $t + 1$  as

$$C_{i,t+1} = \lambda_{t+1}^C \left( \widehat{f}_{i,t+1} - f_{i,t+1} \right), \quad 0 \leq \lambda_{t+1}^C \leq 1 \quad (12)$$



169 where  $\lambda_{t+1}^C$  is the percentage support the central bank is willing to grant at  
 170 time  $t + 1$ . The higher the percentage, the bigger the support of the central  
 171 bank to the unit. A positive value of the central bank intervention  $C_{i,t+1}$   
 172 identifies a lending operation to the unit, while a negative value identifies a  
 173 borrowing operation of the central bank from the unit.

174 We define the maximum liquidity buffer the central bank is willing to  
 175 grant to units in each time  $t$  as  $L_t$ . It describes the maximum net intervention  
 176 of the central bank in the market at time  $t$ . In other words, banks can borrow  
 177 money from the central bank up to  $L_t$ ,  $\forall t = 1, \dots, S$  where  $S$  represents  
 178 the number of time periods of the entire system. We expect that liquidity  
 179 needs from the economic units up to  $L_t$  are fully covered by the central  
 180 bank, while higher needs can only be partially covered up to  $L_t$ . A first  
 181 attempt of liquidity need redistribution we implements here is by describing  
 182 the dynamics of  $\lambda_{t+1}^C$  as follows

$$\lambda_{t+1}^C = \begin{cases} 1 & \text{if } \sum_{i,t+1} [\widehat{f}_{i,t+1} - f_{i,t+1}] \leq L_{t+1}, \\ \frac{L_{t+1}}{\sum_{i,t+1} [\widehat{f}_{i,t+1} - f_{i,t+1}]} & \text{otherwise.} \end{cases} \quad (13)$$

183 Values of  $L_{t+1}$  in (13) greater than the sum of all liquidity needs from the  
 184 units (after the matching mechanism) means that the central bank is willing  
 185 to offer enough liquidity buffer to satisfy the entire system. Therefore, the  
 186 value of  $\lambda_{t+1}^C$  will be equal to 1, meaning that the central bank will give 100%  
 187 support to the units in reaching their liquidity targets. However, values of  
 188  $L_{t+1}$  below the liquidity need of the market represents a situation in which  
 189 the central bank does not have enough liquidity to fully support the units.  
 190 In that case,  $\lambda_{t+1}^C$  will be set up as the ratio between  $L_{t+1}$  and the overall  
 191 liquidity need, that is the partial support that the central bank is able to  
 192 provide ( $\lambda_{t+1}^C < 1$ ).<sup>6</sup>

193 Finally, we can easily write the equation describing the dynamics of the  
 194 liquidity index as

$$f_{i,t+1} = y_{i,t+1} - e_{i,t+1} + C_{i,t+1}. \quad (14)$$

195 where, as we know from (12), the value of  $C_{i,t+1}$  also depends on the Central  
 196 Bank support.

---

<sup>6</sup>Note that the adjustment of  $\lambda_{t+1}^C$  assures a proportional distribution of liquidity to the units relative to the amount requested.

197 The following example shows a numerical exercise that implements the  
 198 matching mechanism introduced above.

199 *Example*

200 Assume that the bilater matrix  $X_0$  of an economic system with three  
 201 units is given by

$$X_0 = \begin{bmatrix} 0 & 0.5 & 0.3 \\ 0.4 & 0 & 0.3 \\ 0.5 & 0.4 & 0 \end{bmatrix}$$

202 from which units computes their outflows  $e_0 = [0.8 \ 0.7 \ 0.9]'$  and inflows  
 203  $y_0 = [0.9 \ 0.9 \ 0.6]$ . Assuming no central bank intervention at time  $t = 0$ ,  
 204 the vector of the liquidity index at this time  $f_0$  will be

$$f_0 = y_0 - e_0 + C_0 = \begin{bmatrix} 0.9 \\ 0.9 \\ 0.6 \end{bmatrix} - \begin{bmatrix} 0.8 \\ 0.7 \\ 0.9 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.3 \end{bmatrix}$$

205 According to equation (4)<sup>7</sup>, the solvency index of next time period will be  
 206  $f_1^* = [0.16 \ 0.29 \ -0.35]'$  and therefore the desired liquidity index for next  
 207 period will be  $\hat{f}_1 = [0.15 \ 0.25 \ -0.24]'$ .<sup>8</sup>

208 The matching mechanism will split the units into those willing to either  
 209 increase  $y$  or  $e$  by computing the percentage changes  $\tau_1^e$  and  $\tau_1^y$  on the basis  
 210 of the directional change  $z_1$ :

$$z_1 = \begin{bmatrix} 0.052 \\ 0.052 \\ 0.052 \end{bmatrix} \quad \tau_1^e = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \tau_1^y = \begin{bmatrix} 0.058 \\ 0.058 \\ 0.087 \end{bmatrix}$$

211 The values of  $z$  clearly show that all units want to increase their liquidity  
 212 index as all  $z$  entries are positive. Therefore, the only way for the units  
 213 to increase their inflows is to borrow liquidity from the central bank. If  
 214 we assume a full support with unlimited liquidity from the central bank,  
 215 by setting  $\lambda_i^C = 1, \forall i = 1, 2, 3$ , then  $C_1 = [0.052 \ 0.052 \ 0.052]'$ . Here the  
 216 public intervention allows units to fulfill their liquidity needs and realize their  
 217 desired liquidity index, that is  $f_1 = \hat{f}_1 = [0.15 \ 0.25 \ -0.24]'$ .

---

<sup>7</sup>We use  $f_0^* = [0.1 \ 0.1 \ 0.1]'$ ,  $\rho = 0.3$ ,  $r = 0.05$ ,  $\mu_i = 0.2 \ \forall i = 1, 2, 3$  and  $T = 1$

<sup>8</sup>Values have been rounded to 2 decimal places

## 218 2.6. Bankruptcy

219 The last component of the model is a criterion for unit bankruptcy. For  
220 the purpose of this exercise, we will focus on situations of liquidity deficit that  
221 may lead to bankruptcy, meaning an prolonged period of negative liquidity  
222 issue that make the unit insolvent. Therefore, an economic unit is declared  
223 bankrupt if its liquidity index is negative for more than  $b$  consecutive time  
224 steps. The number of time steps  $b$  is then the length of time window that  
225 units have to raise liquidity from the market to avoid bankruptcy. When a  
226 unit is declared bankrupt, all flows from and to the bankrupt unit are set to  
227 zero.

## 228 3. A three units example

229 This section presents the results of a three unit exercise that is based  
230 on the example presented in the previous section. We set  $b = 5$ , which  
231 means that units with negative liquidity indices will have to raise liquidity in  
232 not more than 5 time steps to avoid bankruptcy. We set up three different  
233 liquidity policy scenarios to underline policy implications of market stabil-  
234 ity. For simplicity, we will consider a constant liquidity policy over time,  
235 that is  $L_t = L, \forall t = 1, \dots, S$ . We do not apply any liquidity constraint  
236 and bankruptcy criterion in the first 100 time steps to allow the market to  
237 converge to a stable business cycle.

### 238 3.1. Scenario 1: Complete Central Bank Intervention ( $L \rightarrow \infty$ )

239 The first scenario we present describes an environment with an liquidity  
240 buffer that approaches to infinity held by the central bank at every step.  
241 This means that units can always meet their liquidity needs by borrowing  
242 money from the central bank if the market cannot provide enough liquidity.  
243 As  $L$  approaches infinity,  $\lambda_t^C$  will converge to 1  $\forall t = 1, \dots, S$  which means  
244 that the central bank will always meet the units' need for liquidity in the case  
245 when funds cannot be raised from the market. Figure 1 pictures fluctuations  
246 of liquidity and solvency indices for each unit. On the left hand side, we plot  
247 the time series of the liquidity index  $f_i$  (red) and solvency index  $f_i^*$  for each  
248 unit  $i = 1, 2, 3$ , while on the right hand side we show all combination points  
249 of liquidity and solvency indices of each unit.

250 This scenario represents the business cycle base line for comparison with  
251 the other two scenarios (Sordi and Vercelli, this issue).

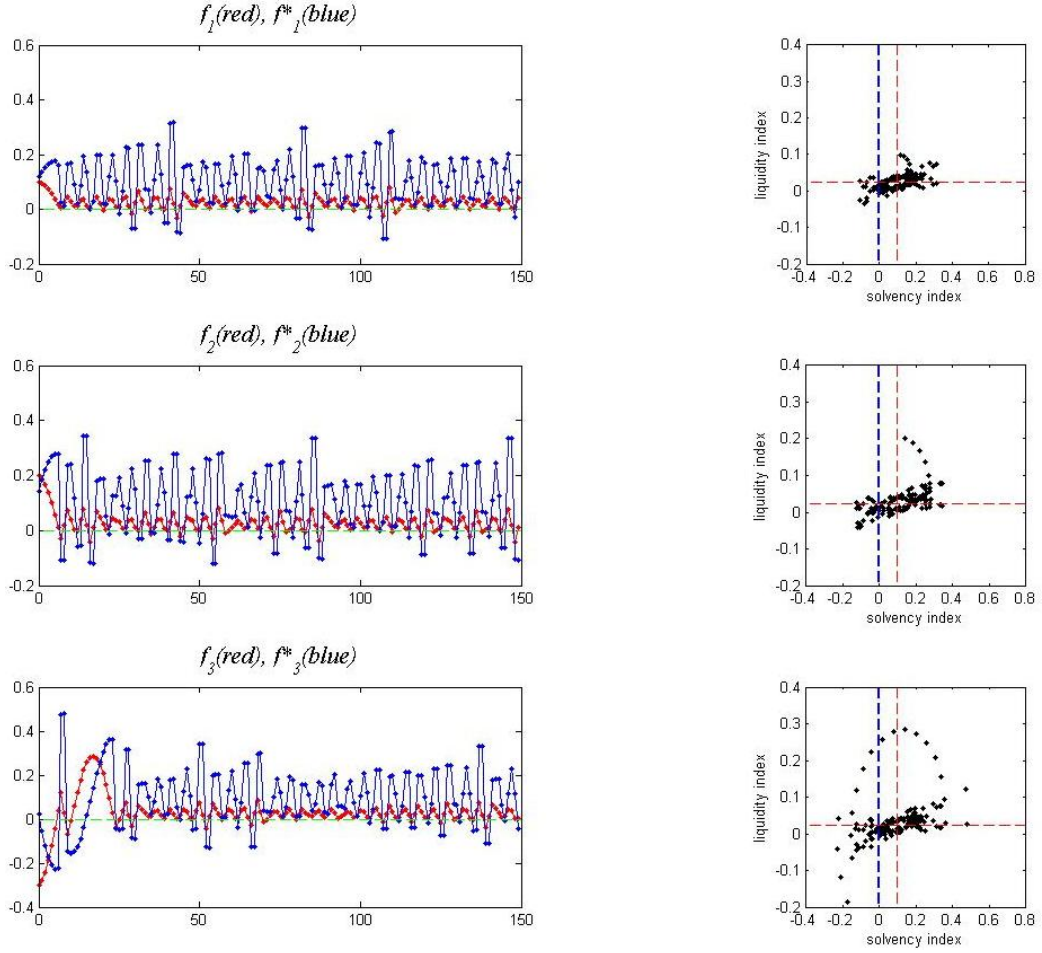


Figure 1: Scenario 1: time series of liquidity (red line) and solvency (blue line) indices for each unit on the left hand side; phase plot of solvency/liquidity indices on the right hand side

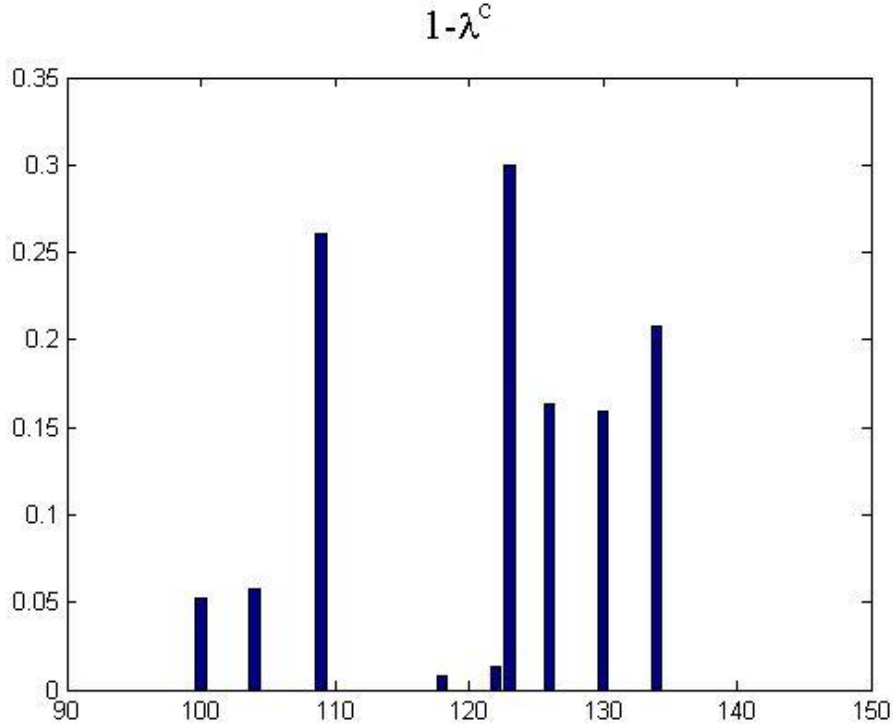


Figure 2: Scenario 2: liquidity deficit (in percentage) of the system as the result of the Central Bank liquidity policy  $L$ .

### 3.2. Scenario 2: Contagion and Counterparty Risk ( $L = 0.1$ )

The second scenario describes an environment with a limited liquidity buffer held by the central bank, which is set at  $L_t = 0.1, \forall t = 1, \dots, S$ . When the market shows an excess of liquidity demand, the central bank can only supply the market with a maximum amount of  $L$ . This will produce a lack of liquidity in the system.

Figure 2 shows nine cases with lack of liquidity, meaning that  $\lambda^C < 1$  from equation (13). The value  $1 - \lambda^C$  can be interpreted as percentage liquidity deficit in the market as a result of the central bank liquidity policy. Note that three major peaks of liquidity lacking greater than 20% at time 109, 123 and 134 respectively can be observed, with an average of 13.6% across all events. The impact of this liquidity policy is pictured in Figure 3, where

The 22% liquidity deficit faced by the units at time 134 caused unit 3 to go bankrupt, which occurs at time 139. Unit 3 was the only unit facing a

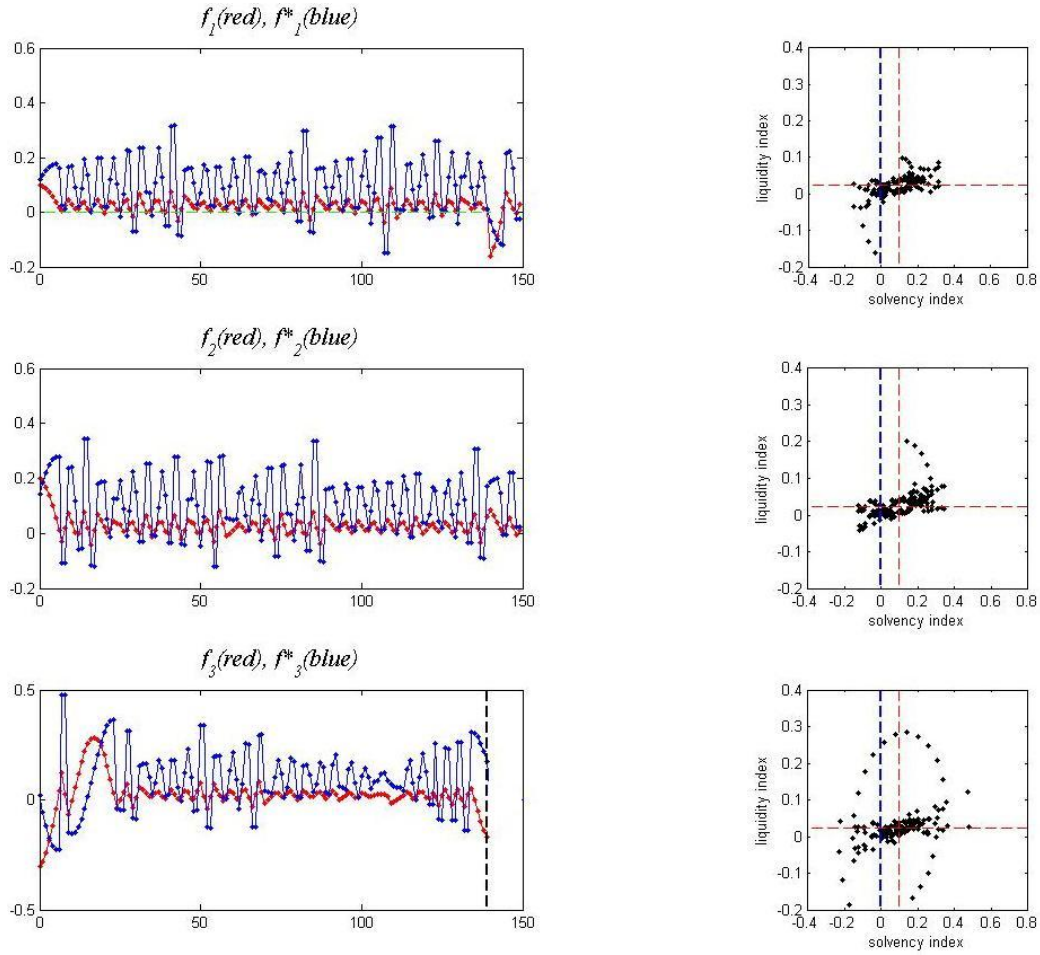


Figure 3: Scenario 2: time series of liquidity (red line) and solvency (blue line) indices for each unit on the left hand side; phase plot of solvency/liquidity indices on the right hand side

negative liquidity index at the time of the event (time 134). The default of unit 3 spreads to the other units as counterparty risk. Unit 2 had a positive exposure to unit 3, that is outflows to unit 3 greater than inflows from unit 3 ( $x_{2,3} > x_{3,2}$ ); this event triggered a positive jumps in the unit 2 liquidity index as it faced a net loss in its outflows. Unit 1 had an opposite effect as it was negatively exposed to unit 3. This bankruptcy event pushed unit 1 to more exposed positions as a result of a negative jump its its liquidity index. However, unit 1 was able to raise liquidity within 5 time steps and avoid bankruptcy.

The last scenario, with a more rigid monetary policy, will describe a different outcome for this unit.

### 3.3. Scenario 3: Contagion and Systemic Risk ( $L = 0.08$ )

In the last scenario we reduce further the maximum liquidity buffer  $L$  to 0.08.

As shown in Figure 4, the lack of systemic liquidity is more substantial than in the previous scenario (average loss of 19.2%). The last and biggest lack in liquidity the units faced at time 131 (32% loss) pushed unit 3 to bankruptcy at time 135. Consequently, the negative exposure of unit 1 to unit 3 results in a downward jump of its liquidity index that dictated its bankruptcy at time 140 (see Figure 5).

With two units bankrupt out of three, the entire market collapses. The very restrictive liquidity policy tested in this scenario eventually triggers a domino effect that results in the collapse of the system.

## 4. Simulation results: endogenous fluctuations and bankruptcy

In this section we extend the three unit example by having a large economic system, with number of units  $N = 100$ . Our aim is to understand the implications of one aspect of the monetary policy strategies, that is the the impact of the systemic liquidity constraint  $L$ , to the spread of contagion and systemic risk.

The initialization of the parameters related to the units behavior are the following<sup>9</sup>:  $\mu = 0.2$ ,  $b = 0.02$ ,  $\rho_e = 0.2$ ,  $\rho_r = 0.08$ ,  $\alpha = 0.25$ .

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<sup>9</sup>The magnitude of the parameters have been chosed according to the simulation experiments presented in Sordi and Vercelli (this issue).

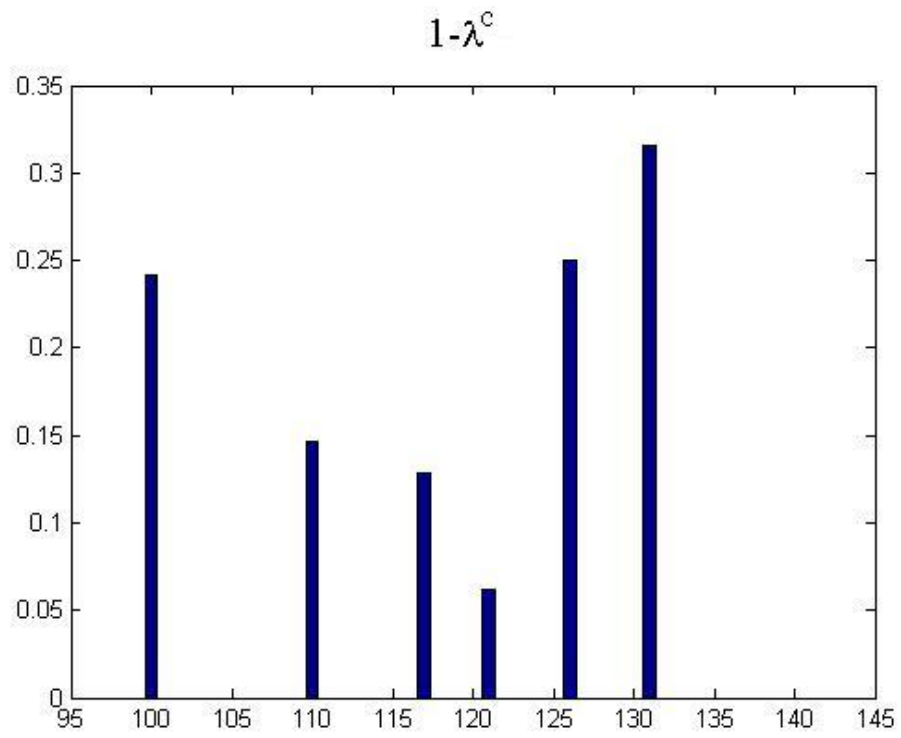


Figure 4: Scenario 3: liquidity deficit (in percentage) of the system as the result of the Central Bank liquidity policy  $L$ .



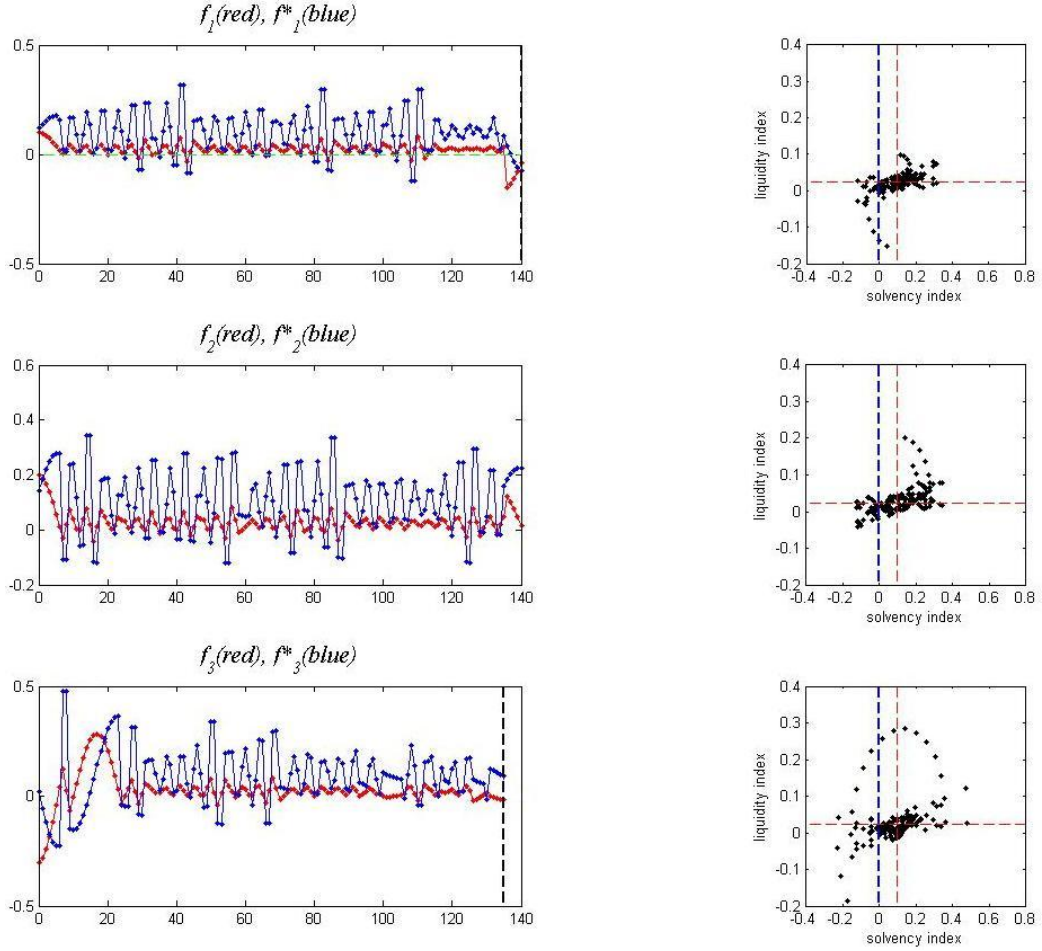


Figure 5: Scenario 3: time series of liquidity (red line) and solvency (blue line) indices for each unit on the left hand side; phase plot of solvency/liquidity indices on the right hand side

297 Units are allowed to interact, that is to transfer monetary flows, to any  
 298 other unit in the market at zero transaction or search costs. Therefore, any  
 299 economic unit can have up to  $N - 1$  counterparties in each time step<sup>10</sup>.

300 We initialize the matrix  $X$  of bilateral flows in a random fashion and we  
 301 test the model under different values of  $L$ ,  $r$  and  $T$ .

#### 302 4.1. Determinants of contagion at the system level

303 First of all, we report the number of bankrupt units under different liq-  
 304 uidity policy regimes. To compare the system with a different number of  
 305 units, we define  $\hat{L} = L/N$  as the average amount of liquidity the central  
 306 bank is willing to provide for each unit. Note that  $\hat{L}$  varies in the range  
 307  $[0, 0.1]$ , which means that the max liquidity buffer the central bank can use  
 308 to satisfy unit liquidity needs is  $\hat{L}xN$ .

309 Figure 6 shows the effect of strict liquidity policy regimes to the number  
 310 of bankrupt units due to systemic risk. Indeed, the lower the level of the  
 311 liquidity buffer  $L$ , the larger the number of bankrupt units, up to the extreme  
 312 case with zero liquidity buffer and the complete collapse of the system. The  
 313 direct effect of the liquidity regime on unit behavior is captured by the unit  
 314 expectation index, representing the percentage number of times the units  
 315 adopt extrapolative expectation instead of regressive ones. As we can see in  
 316 Figure 6, larger liquidity buffers drive the units to more exposed positions,  
 317 by using more extrapolative expectations than regressive ones. However,  
 318 the transition from lower to higher levels of  $L$  does not necessarily means  
 319 a monotonic transition from a lower to a higher expectation index value.  
 320 What appears to be very interesting is the transition, which is not linear  
 321 as we would have expected to see. Indeed, by constantly reducing liquidity  
 322 provision  $L$ , the number of defaulting units first increases, then decreases to  
 323 increase again in the proximity of  $L = 0$  (the case with  $T = 4$  in Figure 6).

324 Sordi and Vercelli (this issue) discussed the effect of higher values of  $T$ ,  
 325 that is the discrete time window of units solvency conditions forecast, on the  
 326 fluctuations of the liquidity index. Indeed, higher values of  $T$  amplify the  
 327 fluctuations of the individual liquidity index, making the unit more exposed  
 328 to liquidity distress. This can be observed in Figure 6 with  $T = 4$ . As

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<sup>10</sup>An interesting extension to the model would be to limit the number of counterparties each unit can have. That would allow us to study the contribution of the specific topology of the network flows among economic units to the spread of contagion. As this analysis is beyond the scope of this paper, we will leave it to future work.

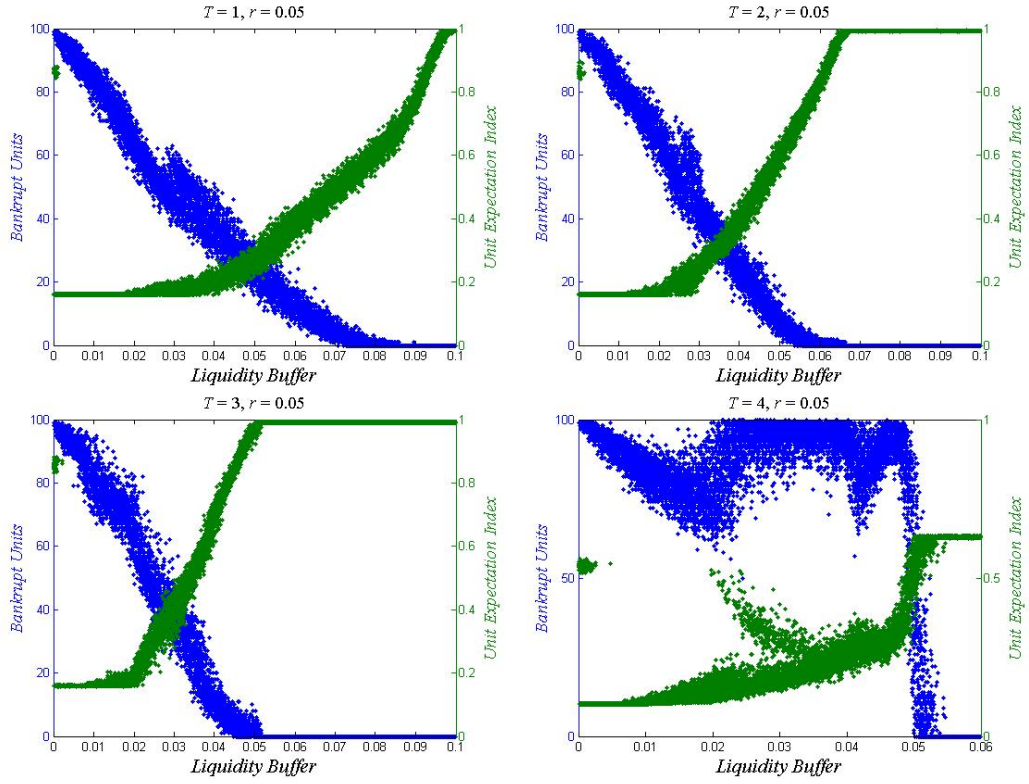


Figure 6: Scatter plots of the number of bankrupt units (blue dots) and the the expectation index (green dots) relative to the monetary policy buffer  $L$  of the system, under different values of  $T$ .

units take more exposed positions, they end up using regressive expectations even with high levels of  $L$  (no lack of liquidity due to limited amount of  $L$ ) just because of their strategy. In this scenario, units seem to have an altered perception of the liquidity policy in operation by the central bank that causes the adoption of extrapolative expectations even with very tight liquidity regimes. The outcome of this altered behavior is a jump in the number of bankrupt units that were not able to realize in time how exposed their position was. They should have recovered very quickly their loss by using regressive expectations. However, the altered perception of market conditions described above does not allow the units to realise their dangerous situation, so increasing their chances of bankruptcy.

#### 4.2. *Determinants of contagion at the individual level*

We are confident that the approach suggested above may help us to understand the individual determinants of bankruptcy of economic units. A preliminary step in this direction is to record the level of liquidity and solvency indices of each bankrupt unit at the time they are affected by either a lack of liquidity due to the liquidity regime of the central bank or the default of one or more counterparties.

Figure 7 plots the combinations of liquidity and solvency values of all bankrupt units when they first were affected by the loss in liquidity (either from the central bank or the counterparty). These plots give us information about the fragility of units along the business cycle. Units are particularly vulnerable to contagion when they have a negative liquidity index. However, the majority of the most exposed points in our simulations is concentrated in the area with positive solvency values around the value  $\mu$ . In other words, the majority of units that went bankrupt because of contagion are the ones that at the time of the contagion had a negative liquidity index and positive solvency index. A tentative interpretation of this *prima facie* surprising result may be that units with a positive solvency react to a worsening of their liquidity index induced by contagion with insufficient energy being reassured by an insolvency index that they still believe to be fairly safe. However the solvency index depends on expectations that deteriorate very rapidly in a period of financial crisis characterized by contagion. In such a situation a positive net worth may very rapidly become negative determining the insolvency of the economic unit. Finally, another result of our simulations is that higher values of  $T$  increase the fluctuations of the liquidity and solvency indices and then amplify the variability of the marginal distribution of the two

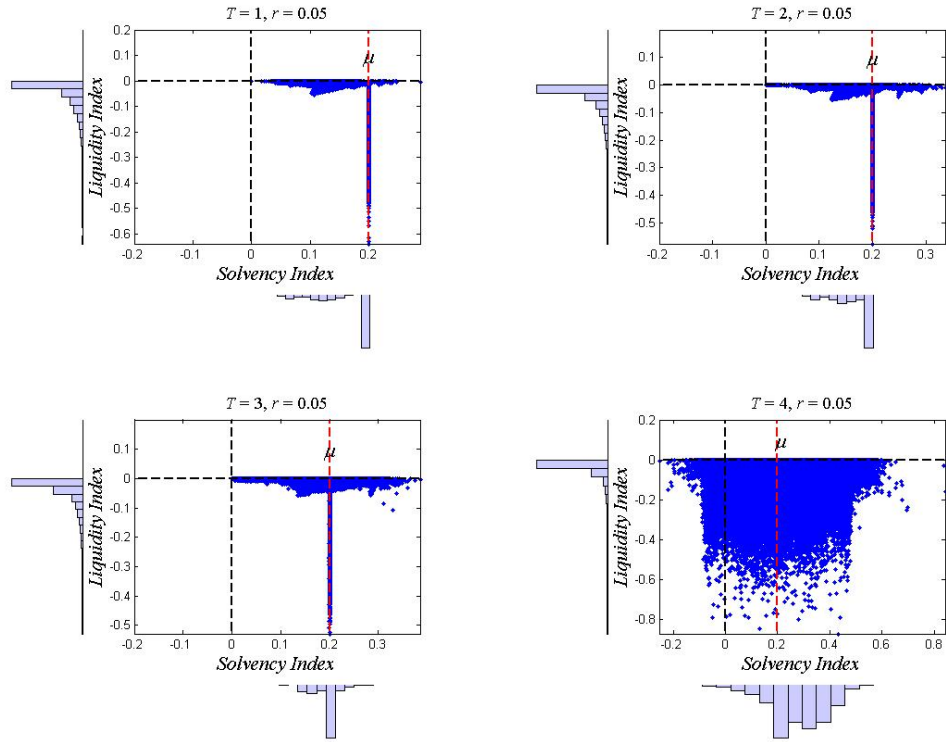


Figure 7: Scatter plot of the combination of liquidity and solvency values of bankrupt units at the time of the contagion. The bar plots at the side and below refer to the marginal histograms of the distribution of the liquidity and solvency values respectively.

366 indices. This result is not surprising since a longer time horizon amplifies  
367 more the effects of a change of expectations contributing to the strength of  
368 financial fluctuations.

369 This phenomenon increases their chance of going bankrupt and explains  
370 the concentration of the points plotted in Figure 7 around the value of  $\mu$ .  
371 Higher values of  $T$  increases the fluctuation of the liquidity and solvency  
372 indices and then amplify the variability of the marginal distribution of the  
373 two indices.

## 374 5. Concluding remarks

375 This work has aimed at a preliminary investigation of the interaction  
376 between liquidity and solvency conditions within systemic risk framework.  
377 Previous research by the authors has shown that this interaction contributes  
378 to the explanation of financial fluctuations for single economic units or for an  
379 agent representative of the entire economy. The aggregate approach of these  
380 contributions, however, clouds some crucial features of financial fluctuations,  
381 such as contagion, that critically depend on the complementarity between  
382 the balance sheets of economic units. This relation brings about a second  
383 order interaction between liquidity and solvency conditions at the level of  
384 the entire economy that depends on their interaction at the units level. This  
385 increases the degree of complexity of the economic dynamics that character-  
386 izes a financialized economy. The simulations performed in the second part  
387 of the paper suggest preliminary results on how financial contagion is trans-  
388 mitted throughout the economy and clarify under which financial conditions  
389 the units are more likely to be affected. The complex dynamics of this model  
390 economy increases the financial fragility of the illiquid economic units even  
391 if their solvency index looks satisfactory in the light of current expectations.

392 However, the significance of the results of the current model is limited by  
393 a series of simplifying assumptions that we aim to relax in future works. In  
394 particular, we refer to the homogeneity of economic units reaction funtions  
395 which follows from setting, as we have done in basic dynamic equations as (7)  
396 and (12), the parameters equal across all units. As for policy implications, a  
397 more comprehensive analysis requires the introduction of public expenditure.

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